3.17. Semantic Concepts

"To be or not to be – that is not a question but a tautology."
Hans Reichenbach, **The Rise of Scientific Philosophy**, p. 250

1. Logical Equivalence and Logical Meaning. We noted earlier that two sentences are **logically equivalent** when both sentences have the same truth table. So the formal sentences "P" and "~~P" are logically equivalent.

P	~P	~~P
1	0	**1**
0	1	**0**

That's significant because the English counterparts to those sentences – say, "It rained yesterday" and "It didn't fail to rain yesterday" – seem to make the same claim, to **mean the same thing**. Logically equivalence provides a formal test of **sameness of meaning** for different logical forms.

But here again it's important to distinguish between **logical form** and **subject matter**. While truth tables are useful for judging whether two English sentences have the same *formal* meaning, they can't tell us whether different English *subject matter* sentences mean the same thing. For to compare the meanings of English sentences, using truth tables, we must first translate those sentences into Formalese. Yet the first step of that translation process will be deciding whether the subject matter sentences get the same sentence letter or not – based on judgments about whether those subject matter sentences *mean the same thing*. By the time truth tables arrive on the scene, sameness of meaning for the subject matter sentences is already settled.

For example, if we wonder whether the English sentences "It rained yesterday" and "It did rain yesterday" mean the same thing, it's no use asking truth tables. If we translate both sentences as "P," we thereby decide that they *do* mean the same thing – and truth tables will just mirror that decision. (Certainly "P" gets the same truth table as "P"; so truth tables judge the two English sentences logically equivalent.) Whereas if we assign the two sentences *different* sentence letters, we're declaring that they *don't* mean the same thing – and truth tables will reflect that decision. (Certainly different sentences letters – say, "P" and "Q" – will, together, get different truth tables.)

Hence we distinguish between different components of meaning: the **formal aspect of meaning** (which truth tables judge), and the **subject matter aspect** (which, for English sentences, lies outside the jurisdiction of truth tables.) Truth tables are useful to us only for deciding on sameness of formal meaning – or, as we will also call it, **logical meaning**.

That resolves what might seem like inconsistent translation practices. On the one hand the 'x-ray translation method' insists that when translating form phrases we proceed mechanically, without considering what clusters of them *mean* – whether, for instance, pairs of negation phrases 'cancel out' in terms of meaning. On the other hand translation *does* require us to stop and think about meaning: when assigning sentence letters, to subject matter sentences. Appreciating the difference between logical and subject matter meaning, we see that in fact there's no inconsistency here: the x-ray translation method – translating without appeal to meaning – applies only to *form* phrases of English, which carry *logical* meaning. And calculating the *logical* meaning of sentences is left to our formal semantics.

The following sentences offer a more interesting illustration of logical meaning.

(1) $\sim(P \wedge Q)$
(2) $(\sim P \wedge \sim Q)$

Though built from the same sentence letters and connectives, the two sentences have different truth tables — and thus *differ* in logical meaning.

P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P \wedge \sim Q)$
1	1	1	**0**	0	0	**0**
1	0	0	**1**	0	1	**0**
0	1	0	**1**	1	0	**0**
0	0	0	**1**	1	1	**1**

English counterparts suggest that these truth tables are right.

(1) $\sim(P \wedge Q)$ (E1) We're not having *both* ice cream *and* cake
(2) $(\sim P \wedge \sim Q)$ (E2) We're not having ice cream and we're not having cake

If (E2) is true, we're not having *either* dessert; while (E1), denying only that we're having *both*, leaves open the possibility of having one. So (E1) and (E2) do make different claims, and *mean* different things.

For a sentence equivalent to (E1) we instead need (E3): if we're not having both, that means we're going without one or the other.

(1) $\sim(P \wedge Q)$ (E1) We're not having *both* ice cream *and* cake
(3) $(\sim P \vee \sim Q)$ (E3) Either we're not having ice cream,
or we're not having cake.

Truth tables agree that these sentences *do* mean the same thing.

			(1)			(3)
P	Q	$(P \wedge Q)$	$\sim(P \wedge Q)$	$\sim P$	$\sim Q$	$(\sim P \vee \sim Q)$
1	1	1	**0**	0	0	**0**
1	0	0	**1**	0	1	**1**
0	1	0	**1**	1	0	**1**
0	0	0	**1**	1	1	**1**

(E4) is an English sentence meaning the same as (E2).

(2) $(\sim P \wedge \sim Q)$ (E2) We're not having ice cream and we're not having cake
(4) $\sim(P \vee Q)$ (E4) We're having *neither* ice cream *nor* cake.

Again truth tables bear this out.

			(4)			(2)
P	Q	$(P \vee Q)$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$(\sim P \wedge \sim Q)$
1	1	1	**0**	0	0	**0**
1	0	1	**0**	0	1	**0**
0	1	1	**0**	1	0	**0**
0	0	0	**1**	1	1	**1**

This pair of equivalences is traditionally called **DeMorgan's Law**.[1]

DeMorgan's Law

"$\sim(P \wedge Q)$" **is equivalent to** "$(\sim P \vee \sim Q)$"

"$\sim(P \vee Q)$" **is equivalent to** "$(\sim P \wedge \sim Q)$"

DeMorgan's Law offers a striking example of how logical equivalence in truth tables reflects intuitive sameness of meaning in English.

[1] But note that saying "$\sim(P \wedge Q)$" and "$(\sim P \vee \sim Q)$" are logically equivalen t (i.e., **semantically equivalent**) doesn't mean these sentences are **equivalent in terms of construction**: the sentences "$\sim(P \wedge Q)$" and "$(\sim P \vee \sim Q)$" have quite different construction trees. Likewise "P" and "~~P" are logically (semantically) equivalent, but they obvious have different construction trees.

2. Tautology and Contradiction. In our survey of translation we already encountered peculiar sentences such as sentence (E5).

(E5) Rex passed Chemistry unless he didn't.

As a report on Rex's Chemistry performance, (E5) is thoroughly uninformative. And precisely because it stakes no claim one way or the other, it cannot fail to be true.

Truth tables agree with our English intuitions: there is no valuation where this disjunction is false (since there's no valuation where both its parts are false).

P: Rex passed Chemistry

(5) $\mathbf{(P \vee {\sim}P)}$

P	**~P**	$\mathbf{(P \vee {\sim}P)}$
1	0	**1**
0	1	**1**

A formal sentence true in every valuation is called a **tautology**, or **logical truth**.

By contrast, an English sentence such as (E6) seems patently absurd.

(E6) Rex passed Chemistry without passing Chemistry

It seems there's no *possible* way (E6) could be true. And truth tables bear this out.

(6) $\mathbf{(P \wedge {\sim}P)}$

P	**~P**	$\mathbf{(P \wedge {\sim}P)}$
1	0	**0**
0	1	**0**

A sentence false in every valuation is a **contradiction**, or **logical falsehood**. Contradictions are the mirror image of tautologies: whereas tautologies seem to assert nothing, a contradiction says too much – more than could *possibly* be true.

When a valuation makes a sentence true we say the valuation **satisfies** that sentence; and a sentence true in at least one valuation is **satisfiable**. So a tautology is satisfiable; but so are sentences such as "P," "$(P \vee Q)$," and "$(P \wedge Q)$". Indeed, the only sort of sentence which *isn't* satisfiable is a contradiction. This makes clear that "satisfiability" is just a more technical name for **consistency**.

3. Consistency and Inconsistency. Consistency can be extended from single sentences to a whole family of sentences, by way of satisfiability: if, for a given set of sentences, there's at least one valuation making **all** those sentences true, the set is **simultaneously satisfiable**. The set of sentences $\{P, (P \vee Q), (P \wedge Q)\}$ is simultaneously satisfiable, since there's a valuation satisfying all three sentences.

P	Q	**$(P \vee Q)$**	**$(P \wedge Q)$**
1	1	**1**	**1**
1	0	1	0
0	1	1	0
0	0	0	0

A simultaneously satisfiable set of sentences is **consistent**. And a set which is *not* simultaneously satisfiable is **unsatisfiable**, or **inconsistent**. The set $\{P, {\sim}P\}$ is inconsistent, since no one valuation *simultaneously* satisfies both sentences.

P	**~P**
1	0
0	1

That is so even though "P" and "~P" are themselves each consistent (satisfiable).

4. Properties of Consistency and Inconsistency. We could state that point in terms of sets. The last example shows that even though a set of sentences is consistent, adding further sentences isn't guaranteed to yield a (bigger) consistent set. The one-sentence set {P} is perfectly consistent; but adding the sentence "~P" yields the inconsistent set {P, ~P}. Indeed, given any consistent set of sentences, we can easily build a bigger inconsistent set by adding the negation of some sentence in the original. We thus say that **consistency doesn't flow uphill** (i.e., moving from smaller set to bigger set by way of added sentences).

But starting with a consistent set of sentences, any smaller set we get from it by throwing out sentences will also be consistent. So we said {P, (P ∨ Q), (P ∧ Q)} is a consistent set. And any smaller set of sentences we get by throwing out sentences – sets such as {P, (P ∨ Q)} or {(P ∨ Q), (P ∧ Q)} or {P} – is also consistent. We say that **consistency flows downhill**.

Predictably, matters are reversed for inconsistency. Starting with the inconsistent set {P, ~P} and throwing out on one or more sentences is not guaranteed to yield a smaller inconsistent set: both {P} and {~P} are consistent sets. Hence **inconsistency doesn't flow downhill**. But beginning with the inconsistent set {P, ~P} and adding more sentences is guaranteed to yield a (larger) inconsistent set – e.g., {P, ~P, Q} or {P, ~P, (P ∨ ~P)}. So **inconsistency flows uphill**.

Summary: Semantic Concepts

- Two sentences are **logically equivalent** when (and only when) they have the same truth tables (i.e., are true in the same valuations, and false in the same valuations).
- A sentence is a **tautology** (or **logical truth**) if it is true in every valuation.
- A sentence is a **contradiction** (or **logical falsehood**) if it is false in every valuation.
- A sentence is **consistent** (or **satisfiable**) if at least one valuation makes it true (**satisfies** it).

- A set of sentences is **consistent** (or **simultaneously satisfiable**) just in case at least one valuation makes every sentence in the set true (**simultaneously satisfies** all those sentences).
- A set of sentences is **inconsistent** (or **unsatisfiable**) if no valuation makes every sentence in the set true (simultaneously satisfies those sentences).

- **Consistency flows downhill, but not uphill**. That is: if from consistent set **S** a larger set **S+** is built by adding sentences, **S+** is not guaranteed to be consistent. But if from consistent set **S** a smaller set **S-** is built by throwing out some sentence(s), **S-** is guaranteed to be consistent.

- **Inconsistency flows uphill, but not downhill**. That is: beginning with inconsistent set **S**, adding further sentences to it yields a larger set **S+** guaranteed to be inconsistent. But if from inconsistent set **S** a smaller set **S-** is built by throwing some sentence(s) out of **S**, **S-** is not guaranteed to be inconsistent.